From Γ to A^w

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I need to better understand the relationship between Γ-calculus and its parent, A^w.

$$\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\beta]{m^{ab}_c} \left(\begin{array}{c|cc} \mu\omega & c & S \\ \hline c & \gamma + \alpha\delta/\mu & \epsilon + \delta\theta/\mu \\ S & \phi + \alpha\psi/\mu & \Xi + \psi\theta/\mu \end{array} \right)_{T_a, T_b \to T_c}$$

(col sum is 1)

$$R^{\pm}_{ab} \underset{\gamma}{=} \begin{array}{c|cc} 1 & a & b \\ \hline a & 1 & 1 - T_a^{\pm 1} \\ b & 0 & T_a^{\pm 1} \end{array}$$

β calculus (col sum is $\sigma_x - 1$)

$$\begin{array}{c|c} \omega & H \\ \hline u & \alpha \\ v & \beta \\ T & \Xi \end{array} \xrightarrow[T_u, T_v \to T_w]{\beta::tm^{uv}_w} \begin{array}{c|c} \omega & H \\ \hline w & \alpha + \beta \\ T & \Xi \end{array} \qquad \begin{array}{c|ccc} \omega & x & y & H \\ \hline T & \alpha & \beta & \Xi \end{array} \xrightarrow[\beta]{hm^{xy}_z} \begin{array}{c|cc} \omega & z & H \\ \hline T & \alpha + \sigma_x\beta & \Xi \end{array}$$

$$\begin{array}{c|cc} \omega & x & H \\ \hline u & \alpha & \theta \\ T & \phi & \Xi \end{array} \xrightarrow[\nu:=1+\alpha]{\beta::tha^{ux}} \begin{array}{c|cc} \nu\omega & x & H \\ \hline u & \sigma_x\alpha/\nu & \sigma_x\theta/\nu \\ T & \phi/\nu & \Xi - \phi\theta/\nu \end{array} \qquad \rho^{\pm}_{ux} \underset{\beta}{=} \begin{array}{c|c} 1 & x \\ \hline u & T_u^{\pm 1} - 1 \end{array}$$

Old β, as in KBH paper:

$$tm^{uv}_w: \begin{array}{c|c} \omega & H \\ \hline u & \alpha \\ v & \beta \\ T & \gamma \end{array} \mapsto \left(\begin{array}{c|c} \omega & H \\ \hline w & \alpha+\beta \\ T & \gamma \end{array}\right) /\!\!/ (t_u, t_v \to t_w), \qquad \zeta^\beta(\rho^\pm_{ux}) = \begin{array}{c|c} 1 & x \\ \hline u & t_u^{\pm 1} - 1 \end{array}.$$

$$hm^{xy}_z: \begin{array}{c|ccc} \omega & x & y & H \\ \hline T & \alpha & \beta & \gamma \end{array} \mapsto \begin{array}{c|cc} \omega & z & H \\ \hline T & \alpha+\beta+\langle\alpha\rangle\beta & \gamma \end{array},$$

$$tha^{ux}: \begin{array}{c|cc} \omega & x & H \\ \hline u & \alpha & \beta \\ T & \gamma & \delta \end{array} \mapsto \begin{array}{c|cc} \omega(1+\alpha) & x & H \\ \hline u & \alpha(1+\langle\gamma\rangle/(1+\alpha)) & \beta(1+\langle\gamma\rangle/(1+\alpha)) \\ T & \gamma/(1+\alpha) & \delta-\gamma\beta/(1+\alpha) \end{array},$$

β_0, as in KBH paper:

e.g., tha:

– As dictated by Equation (31), ω is replaced by $\omega + \log\left(1 + \frac{e^{c_\gamma}-1}{c_\gamma} c_u \gamma_u\right)$.

– As dictated by Equations (24) and (28), every column $\alpha = \begin{pmatrix} \alpha_u \\ \alpha_{\text{rest}} \end{pmatrix}$ in A (including the column γ itself) is replaced by

$$\left(1 + c_u \gamma_u \frac{e^{c_\gamma}-1}{c_\gamma}\right)^{-1} \begin{pmatrix} e^{c_\gamma}\alpha_u \\ \alpha_{\text{rest}} - c_u \frac{e^{c_\gamma}-1}{c_\gamma} (c\gamma)_{\text{rest}} \end{pmatrix},$$

where $(c\gamma)_{\text{rest}}$ is the column whose row v entry is $c_v\gamma_v$, for any $v \neq u$.

$A^w(H;T)$ calculus

$A^w(S)$ calculus