

# From $\Gamma$ to $A^w$

June-01-15 8:37 AM

I need to better understand the relationship between  $\Gamma$ -calculus and its parent,  $A^w$ .

$$\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\beta]{m_c^{ab}} \left( \begin{array}{c|cc} \mu\omega & c & S \\ \hline c & \gamma + \alpha\delta/\mu & \epsilon + \delta\theta/\mu \\ S & \phi + \alpha\psi/\mu & \Xi + \psi\theta/\mu \end{array} \right)_{T_a, T_b \rightarrow T_c}$$

(col sum is 1)

$$R_{ab}^{\pm} = \frac{1}{\gamma} \begin{array}{c|cc} & a & b \\ \hline a & 1 & 1 - T_a^{\pm 1} \\ b & 0 & T_a^{\pm 1} \end{array}$$

$\beta$  calculus

(col sum is  $\sigma_x - 1$ )

$$\begin{array}{c|c} \omega & H \\ \hline u & \alpha \\ v & \beta \\ T & \Xi \end{array} \xrightarrow[T_u, T_v \rightarrow T_w]{\beta::tm_w^{uv}} \begin{array}{c|c} \omega & H \\ \hline w & \alpha + \beta \\ T & \Xi \end{array} \quad \begin{array}{c|ccc} \omega & x & y & H \\ \hline T & \alpha & \beta & \Xi \end{array} \xrightarrow[\beta]{hm_z^{xy}} \begin{array}{c|cc} \omega & z & H \\ \hline T & \alpha + \sigma_x \beta & \Xi \end{array}$$

$$\begin{array}{c|cc} \omega & x & H \\ \hline u & \alpha & \theta \\ T & \phi & \Xi \end{array} \xrightarrow[v:=1+\alpha]{\beta::tha^{ux}} \begin{array}{c|cc} v\omega & x & H \\ \hline u & \sigma_x \alpha / v & \sigma_x \theta / v \\ T & \phi / v & \Xi - \phi \theta / v \end{array} \quad \rho_{ux}^{\pm} = \frac{1}{\beta} \begin{array}{c|c} & x \\ \hline u & T_u^{\pm 1} - 1 \end{array}$$

old  $\beta$ , as in KBH paper:

$$tm_w^{uv}: \begin{array}{c|c} \omega & H \\ u & \alpha \\ v & \beta \\ T & \gamma \end{array} \mapsto \left( \begin{array}{c|c} \omega & H \\ w & \alpha + \beta \\ T & \gamma \end{array} \right) // (t_u, t_v \rightarrow t_w),$$

$$\zeta^\beta(\rho_{ux}^\pm) = \frac{1}{u} \left| \frac{x}{t_u^{\pm 1} - 1} \right|.$$

$$hm_z^{xy}: \begin{array}{c|ccc} \omega & x & y & H \\ T & \alpha & \beta & \gamma \end{array} \mapsto \begin{array}{c|c} \omega & z & H \\ T & \alpha + \beta + \langle \alpha \rangle \beta & \gamma \end{array},$$

$$tha^{ux}: \begin{array}{c|ccc} \omega & x & H \\ u & \alpha & \beta \\ T & \gamma & \delta \end{array} \mapsto \begin{array}{c|cc} \omega(1+\alpha) & x & H \\ u & \alpha(1 + \langle \gamma \rangle / (1 + \alpha)) & \beta(1 + \langle \gamma \rangle / (1 + \alpha)) \\ T & \gamma / (1 + \alpha) & \delta - \gamma\beta / (1 + \alpha) \end{array},$$

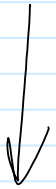
$\beta_0$ , as in KBH paper:

e.g., that:

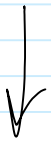
- As dictated by Equation (31),  $\omega$  is replaced by  $\omega + \log \left( 1 + \frac{e^{c\gamma} - 1}{c_\gamma} c_u \gamma_u \right)$ .
- As dictated by Equations (24) and (28), every column  $\alpha = \begin{pmatrix} \alpha_u \\ \alpha_{\text{rest}} \end{pmatrix}$  in  $A$  (including the column  $\gamma$  itself) is replaced by

$$\left( 1 + c_u \gamma_u \frac{e^{c\gamma} - 1}{c_\gamma} \right)^{-1} \begin{pmatrix} e^{c\gamma} \alpha_u \\ \alpha_{\text{rest}} - c_u \frac{e^{c\gamma} - 1}{c_\gamma} (c\gamma)_{\text{rest}} \end{pmatrix},$$

where  $(c\gamma)_{\text{rest}}$  is the column whose row  $v$  entry is  $c_v \gamma_v$ , for any  $v \neq u$ .



$A^W(H;T)$  calculus



$A^W(S)$  calculus